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ABSTRACT

Contingency tables, and their associated statistical tests, are frequently used in educational and social research. Popular statistical tests used in contingency table analyses include the Pearson chi-square test and the likelihood ratio chi-square test. These two tests are chi-square distributed under large sample conditions. However, when a given contingency table includes small cell frequencies the obtained chi-square statistic may not be asymptotically valid, and the associated probability values will be inappropriate. An alternative statistic which has been recommended for small samples or sparse conditions is the power-divergence statistic (Read & Cressie, 1988). This study was an investigation of the performance of three tests of independence for I x J contingency tables under small sample conditions. Specifically, the objectives of the research were to investigate the power and Type I error rates under small sample or sparse conditions of the Pearson chi-square test, the likelihood ratio chi-square test, and the Read and Cressie power-divergence statistic with $\lambda = 2/3$ (Read & Cressie, 1988). The power and Type I error rates were estimated for a variety of table dimensions, marginal distributions, sample sizes, and effect sizes. (Contains 16 references, 4 figures, and 4 tables.) (Author)

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Comparison of Statistical Tests of Independence for Sparse I x J Contingency Tables

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Abstract

Contingency tables, and their associated statistical tests, are frequently used in educational and social research. Popular statistical tests used in contingency table analyses include the Pearson chi-square test and the likelihood ratio chi-square test. These two tests are chi-square distributed, under large-sample conditions. However, when a given contingency table includes small cell frequencies the obtained chi-square statistic may not be asymptotically valid, and the associated probability values will be inappropriate. An alternative statistic which has been recommended for small samples or sparse conditions is the power-divergence statistic (Read & Cressie, 1988).

This study was an investigation of the performance of three tests of independence for I x J contingency tables under small sample conditions. Specifically, the objectives of the research were to investigate the power and Type I error rates under small sample or sparse conditions of the Pearson chi-square test, the likelihood ratio chi-square test, and the Read and Cressie power-divergence statistic with $\lambda = 2/3$ (Read & Cressie, 1988). The power and Type I error rates were estimated for a variety of table dimensions, marginal distributions, sample sizes, and effect sizes.

Comparison of Statistical Tests of Independence for Sparse I x J Contingency Tables

Introduction

Contingency tables, and their associated statistical tests, are frequently used in educational and social research. Popular statistical tests used in contingency table analyses include the Pearson chi-square test of independence and the likelihood ratio chi-square test of independence. These two tests are asymptotically equivalent and, under large-sample conditions, are chi-square distributed. However, when a given contingency table includes small cell frequencies, the obtained chi-square statistic may *not* be chi-square distributed. Under these conditions, when the statistic is not asymptotically valid, the associated probability values will be inappropriate. If an obtained statistic is not distributed according to the chi-square distribution, researchers may mistakenly base decisions on inaccurate information. They may reject a null hypothesis when it is true, or fail to reject a null hypothesis when it is false.

Several statistics have been proposed for testing independence between variables in a two dimensional contingency table. Three of these statistics were investigated in this research.

The Pearson chi-square test is given below, where O refers to the *observed* frequency of each cell and E to the *expected* frequency. The summation is over the cells in the contingency table.

$$\chi^2 = \sum_{i=1}^I \frac{(O_i - E_i)^2}{E_i}$$

The likelihood ratio chi-square test is asymptotically equivalent to Pearson's chi-square test (Fienberg, 1977), and is often used with larger contingency tables, because of its facility with model testing and chi-square partitioning (Upton, 1978). This statistic is computed as:

$$G^2 = 2 \sum_{i=1}^c O_i \ln \left(\frac{O_i}{E_i} \right)$$

Various corrective actions may be taken by researchers when contingency tables have small overall samples or some sparse cells. One corrective action is to collapse across categories of one or more of the research variables. However, this may lead to categories of lesser meaning than the original variable held. A second action may be to use exact methods to compute the probability for a given table, rather than to base probability on the tabled chi-square distribution (Agresti, 1990). For larger I x J tables, this can be computationally prohibitive. Another recommended course of action has been the use of alternative statistics (e.g., Kroll, 1989; Richardson, 1990). Read and Cressie (1988) have suggested the following statistic as one which is less susceptible to the effects of sparseness than either X^2 or G^2 .

$$RC = \frac{2}{\lambda(\lambda+1)} \sum_{i=1}^c O_i \left[\left(\frac{O_i}{E_i} \right)^\lambda - 1 \right]$$

In their unified model, Cressie and Read (1984), and Read and Cressie (1988), suggest that in fact Pearson's chi-square (with λ set equal to 1) and the likelihood ratio chi-square (the limit as λ approaches 0) are just two members of a family of *power-divergence* statistics, all of which can be represented by the formula above. Read and Cressie (1988) have found the power-divergence statistic with $\lambda = 2/3$ to have some excellent properties, including optimal performance under small sample conditions.

Purpose

The Type I error estimates for small sample chi-square statistics (both X^2 and G^2) have been investigated for 2 x 2 tables (Camilli & Hopkins, 1978, 1979; Larntz, 1978; Roscoe & Byars, 1971; Thompson, 1988). Considerably less research has been conducted on relative power, or on tables of larger dimensions (Mehta & Hilton, 1993; Parshall & Kromrey, 1994). However, it is often the case that as table size increases, so does the probability of at least some very low frequency cells (Read & Cressie, 1988).

The purpose of this study was to investigate the performance of three tests of independence for I x J contingency tables under small samples and sparse conditions. Specifically, the objectives of the research to be reported were to investigate the power and Type I error rates under small sample and sparse conditions of the Pearson chi-square test, the likelihood ratio chi-square test, and the Read and Cressie statistic with $\lambda = 2/3$. The power and Type I error rates were estimated for various table dimensions, marginal distributions, sample sizes, and effect sizes.

Educational Importance of the Study

Because of the frequent application in educational research of contingency table analyses, the operating characteristics of statistical tests of independence is an important area of inquiry. Similarly, the use of small samples is often necessary in educational research. Knowledge of the power and expected Type I error rates of tests of contingency tables in which small samples are involved (either for the total table or for some subset of cells) will help inform researchers who are planning studies in which contingency table analyses will be used.

Method

The research to be reported was a Monte Carlo study. Random samples were generated for a series of I x J contingency tables: 2 x 5, 3 x 5, 5 x 5, and 2 x 7, 3 x 7, 5 x 7. For each table dimension, three conditions of population marginal distributions were examined: equal marginals, slightly skewed, and highly skewed. Small, medium, and large population effect sizes were examined. In Cohen's (1988) power analysis text, these effect sizes correspond to w values of 0.10, 0.30, and 0.50, respectively. Cohen's effect size w is computed as:

$$w = \sqrt{\sum \frac{(P_{li} - P_{oi})^2}{P_{oi}}}$$

Where P_{0i} is the expected proportion in cell i , and P_{1i} is the observed proportion in cell i . In addition to these effect sizes, a null model (no effect, or no dependency between the two marginal variables) was also examined, to evaluate the extent to which Type I error rates match nominal alpha levels.

Programming for the Monte Carlo Study

The programming for the Monte Carlo study was written in SAS version 6.06. The data were generated using uniform random numbers on the zero to one interval (the SAS RANUNI function). To simulate samples for a test of independence, a separate series of random numbers was generated for each row of the contingency table, with each row consisting of the same number of observations. The observations were then assigned to columns in the table based upon the value of the random number.

For example, with a 2 X 5 table with equal marginals and an effect size of zero, two series of random numbers were generated. Observations with random numbers between zero and .20 were assigned to the first column of the contingency table, those with random numbers between .20 and .40 were assigned to the second column, etc. This procedure yields tables in which the expected proportion in each cell is .10, each column marginal proportion is expected to be .10, and each row marginal proportion is fixed at .50.

The column marginal proportions of the tables examined in the study were controlled by assigning larger or smaller ranges of the uniform random numbers to each column. For example, for a 2 X 5 table with 60:10:10:10:10 column marginals and an effect size of zero, observations with random numbers between zero and .60 were assigned to the first column of the contingency

table, those with random numbers between .60 and .70 were assigned to the second column, those with random numbers between .70 and .80 were assigned to the third column, those with random numbers between .80 and .90 were assigned to the fourth column, and those with random numbers higher than .90 were assigned to the fifth column. As with the equal marginals procedure described above, the tables produced with this procedure have cells with expected proportions that are equal to the products of the table's marginal proportions.

Three column marginal distributions were examined in this study. The equal marginal condition provided equal proportions at each level of the column variable. A slightly skewed marginal distribution was produced by generating tables in which the expected value of the first column of the contingency table was 60% of the data, and the remaining 40% was evenly dispersed over the other columns of the table (i.e., for a five-level column variable each level was expected to receive $40/4$ or 10% of the observations; for a seven-level column variable each level was expected to receive $40/6$ or approximately 6.67% of the data). Similarly, a more highly skewed column marginal was produced by generating tables in which the expected value of the first column was 80% of the data and the remaining 20% was evenly distributed over the remaining columns.

Finally, non-null effects were generated by assigning observations to table cells in proportions that differed from the products of the table's marginal proportions. For example, for a 2 X 5 table with equal marginals and an effect size of .50, observations from the first row with random numbers between zero and .312 were assigned to the first column of the contingency table, those with random numbers between .312 and .624 were assigned to the second column, those with random numbers between .624 and .712 were assigned to the third column, those with random

numbers between .712 and .800 were assigned to the fourth column, and those with random numbers larger than .800 were assigned to the fifth column. The procedure was reversed for the random numbers representing the second row of the table. For these data, random numbers between 0 and .088 were assigned to the first column, those with random numbers between .088 and .176 were assigned to the second column, those with random numbers between .176 and .488 were assigned to the third column, those with random numbers between .488 and .800 were assigned to the fourth column, and those with random numbers higher than .800 were assigned to the fifth column. This procedure yields tables in which the expected proportion in each cell of the first four columns deviates by .056 from the product of the marginal proportions, corresponding to Cohen's (1988) w of .50.

For each size of contingency table investigated, a total of five sample sizes were produced. For the 2 X J tables, overall sample sizes of 16, 32, 64, 128, and 256 were studied. For the 3 X J tables, sizes were 18, 33, 66, 129, and 258. For the 5 X J tables, the sample sizes investigated were 15, 30, 65, 130, and 255. For each table size and sample size, the total sample size was equally divided over the rows in the contingency table. Five thousand samples of each size were drawn under each of the experimental conditions examined. The use of five thousand replications provide maximum 95% confidence intervals of $\pm .014$ around the observed proportion of null hypotheses rejected (Robey & Barcikowski, 1992).

For each condition, three test statistics were computed: (a) Pearson's chi-square test of independence, (b) the likelihood ratio chi-square test, (c) and the Read and Cressie power-

divergence statistic with $\lambda = 2/3$. Estimates of the statistical power of each test were conducted at alpha levels of .01, .05, and .10. (However, to limit the number of figures included, only the results for the nominal alpha level of .05 are presented in figures.)

Results

The empirical estimates of the Type I error rates for the three statistical testing procedures are presented in Table 1 for the conditions examined in this study. At a .01 nominal alpha level, both the Pearson chi-square and the Read and Cressie statistic were consistently conservative in Type I error control for the smallest samples examined. For the samples of size 16, the estimated Type I error rates ranged from zero to only .003 for the Pearson chi-square and to .004 for the Read and Cressie statistic. As the sample sizes increased, the estimates of Type I error rates of both procedures approached the nominal alpha level. However, for the conditions involving asymmetric marginal distributions, these tests remained somewhat conservative even with the largest samples examined in this study. For example, with the 3 X 7 tables and samples of size 258, the estimates of Type I error rates were .005 for both of these tests in the extreme asymmetry condition. In contrast to the conservatism shown by the Pearson chi-square and the Read and Cressie statistics, the likelihood ratio chi-square showed a marked tendency to be excessively liberal in Type I error control. Using Bradley's (1975) liberal criterion of statistical robustness (i.e., estimated Type I error rate within the range of $\alpha_{\text{nominal}} \pm .5 \alpha_{\text{nominal}}$), the likelihood ratio chi-square exceeded these limits in 38 of the 90 conditions examined with a nominal alpha

level of .01. The most extreme Type I error rate was seen in the 5 X 7 tables with equal marginals and samples of size 65, in which the estimate of the Type I error rate for the test was .041, more than four times the nominal alpha level.

Insert Table 1 About Here

The results were consistent with nominal alpha levels of .05 and .10, although as should be expected the Type I error rates were closer to nominal levels for these more liberal alpha levels. The Pearson chi-square and the Read and Cressie statistic remained conservative with small samples and asymmetric marginals at both of these nominal alpha levels. The likelihood ratio chi-square remained liberal in many of the conditions examined, exceeding Bradley's liberal limits of robustness in 44 of the 90 conditions examined at a nominal alpha level of .05 and in 47 of the 90 conditions at a nominal alpha level of .10.

Of interest to note in Table 1, the relative conservatism of the Pearson chi-square and the Read and Cressie statistic depended upon the table size, with the Read and Cressie statistic being less conservative than the Pearson chi-square in the smaller tables, but more conservative in the larger tables. For example, at the .05 nominal alpha level in the 2 X 7 tables with equal marginals and samples of size 16, the estimated Type I error rate for the Read and Cressie statistic was .027 while that of the Pearson chi-square was .017. With the smallest sample size in the 3 X 5 and 3 X 7 tables with equal marginals, however, these tests were nearly equally conservative (.032 vs. .033 for the 3 X 5 tables, and .021 vs. .022 for the 3 X 7 tables). In contrast, for the 5 X 5 and 5 X 7 tables with the smallest samples and equal marginals, the

Pearson chi-square was less conservative than the Read and Cressie statistic (.027 vs. .014 for the 5 X 5 tables, and .021 vs. .006 for the 5 X 7 tables). Regardless of table dimensions, however, with larger sample sizes the estimated Type I error rates of these two statistics converged (see Figures 1 and 2).

Insert Figures 1 and 2 About Here

Estimates of statistical power are provided in Tables 2 and 3. Because the likelihood ratio chi-square did not adequately control Type I error rates for the sparse tables examined in this study, power estimates for this test are not included. Table 2 provides power estimates averaged across effect sizes, while detailed results are provided in Table 3.

Insert Tables 2 and 3 About Here

An examination of Table 2 shows that the average power differences between the Read and Cressie statistic and the Pearson chi-square were small. However, the Read and Cressie statistic was consistently more powerful, on average, than the Pearson chi-square for the smaller tables examined, while the Pearson chi-square evidenced power advantages for the larger tables. Such power differences, however, were found only with the smaller sample sizes. With the larger sample sizes examined, the two tests showed similar power across conditions examined. This pattern of power differences is consistent with the pattern of Type I error rate estimates presented

in Table 1, with each test having lower power for those table sizes in which the test is the more conservative in terms of Type I error control.

For example, at the nominal alpha level of .01, the greatest difference favoring the Read and Cressie statistic was obtained in the 2 X 7 tables with samples of size 64 and extreme skewness in the marginals. In this condition, the power of the Read and Cressie statistic was .230, while that of the Pearson chi-square was .179, a difference of .051. The greatest difference favoring the Pearson chi-square was obtained in the 5 X 7 tables with samples of size 130 and extreme marginal skewness. In this condition, the estimated power of the Pearson chi-square was .285, while that of the Read and Cressie statistic was .242, a difference of .043. The Read and Cressie statistic provided slight but consistent power advantages relative to the Pearson chi-square for the 2 X 5 and 2 X 7 tables.

Similarly, for the nominal alpha level of .05, the greatest difference in power estimates favoring the Read and Cressie statistic was seen in the 2 X 7 tables with samples of size 32 and extreme marginal skewness, where the power of the Read and Cressie statistic was .124 while that of the Pearson chi-square was .087, a difference of .037. The greatest difference favoring the Pearson chi-square was obtained in the 5 X 7 tables with samples of size 65 and extremely skewed marginals. In this condition, the estimated power of the Pearson chi-square was .163, while that of the Read and Cressie statistic was .099, a power difference of .064.

Finally, for the nominal alpha level of .10, the greatest power difference favoring the Read and Cressie statistic was seen in the 2 X 7 tables with samples of size 32 and extreme marginal skewness. In this condition the estimated power of the Read and Cressie statistic was .250 while

that of the Pearson chi-square was .203, and a difference of .047. Conversely, the greatest estimated power difference favoring the Pearson chi-square was obtained in the 5 X 7 tables with samples of size 65 and extreme skewness in the marginal distributions. In this condition the power of the Pearson chi-square was .256 while that of the Read and Cressie statistic was .175, a difference of .081.

In an examination of the more detailed results on the statistical power estimates presented in Table 3, it should be noted that for those conditions that showed the greatest overall power differences, the differences increase with the effect size of the condition. For example, at a nominal alpha level of .05, with the 2 X 7 tables with extreme skewness and sample size 32, both tests showed almost no power at the small effect size (.004 for the Pearson chi-square and .006 for the Read and Cressie statistic). For the medium effect size (0.3) the estimated power of the Pearson chi-square was .043 while that of the Read and Cressie was .061. Finally, for the large effect size (0.5) the power estimates were .213 and .304, for the two tests respectively (see Figure 3). Similarly, for the 5 X 7 tables with extreme skewness and samples of size 65, under a nominal alpha of .05, the Pearson chi-square and the Read and Cressie statistic showed power estimates for the small effect size of .022 and .007, respectively. With a medium effect size the power estimates were .099 for the Pearson chi-square and .049 for the Read and Cressie statistic, while for the large effect size the power estimates were .369 and .240 for the two tests (see Figure 4).

Insert Figures 3 and 4 About Here

A summary of the power comparisons is presented in Table 4. This table presents the number and percent of conditions in which the Pearson chi-square was nominally more powerful than the Read and Cressie statistic, the number and percent of conditions in which the Read and Cressie statistic was more powerful, and the number and percent of conditions in which the power estimates were identical. For example, at a nominal alpha level of .01, for the 2 X 5 and 2 X 7 tables, the Read and Cressie statistic was more powerful than the Pearson chi-square in 73% of the conditions examined, and this test was not less powerful than the chi-square in any of the conditions examined. For the 3 X 5 and 3 X 7 tables, the tests showed identical power in 40% of the conditions, the chi-square test was more powerful in 33% of the conditions and the Read and Cressie statistic was more powerful in 27% of the conditions. For the 5 X 5 and 5 X 7 tables, the chi-square test was more powerful than the Read and Cressie statistic in 78% of the conditions. As may be seen in Table 4, this pattern was evident across nominal alpha levels. However, for the 3 X 5 and 3 X 7 tables, the power estimates for the Read and Cressie statistic exceeded those of the Pearson chi-square at nominal alpha levels of .05 and .10 more frequently than it did for the nominal alpha level of .01.

Insert Table 4 About Here

Discussion

The likelihood ratio test demonstrated a poor ability to maintain the nominal Type I error rate, disqualifying it for further consideration. (A related, earlier investigation of 2x2 and 2x3

contingency table analyses found the likelihood ratio test to display relatively poor power along with liberal Type I error rates compared to other statistics investigated. See Parshall & Kromrey, 1994.) The performance of the remaining, competing statistics in this study converged as sample size increased, regardless of the size of the table. On a practical basis, this means that for the conditions studied, once a sample is large enough it may be a matter of indifference to the practitioner whether Pearson's chi-square or the Read and Cressie statistic is used.

With the smaller samples however, differences in the performance of the two statistics were found. The Read and Cressie statistic displayed greater power than Pearson's statistic in the smaller tables, while Pearson's chi-square test out-performed the Read and Cressie statistic in the larger tables. While both tests demonstrated increased power as the effect size increased, this was not a general improvement. Rather, the Read and Cressie statistic displayed a greater increase in power associated with increased effect size in just those smaller tables where it already displayed a performance advantage over Pearson's test, while Pearson's test demonstrated a greater power increase associated with increased effect size in the larger tables where it displayed better performance. Both statistics are more powerful under equal marginals conditions. Both also demonstrate a marked conservativeness in the tables with highly skewed marginals, although the general pattern of better performance of the Read and Cressie for smaller tables and better performance for the Pearson for larger test hold here as well.

Read and Cressie (1988) point out a number of the variables and assumptions which may need to be considered, including "the sample size n , the number of cells k , the form of the null model (loglinear, etc.), and the 'direction of departure' of the alternative from the null" (pg. 80). Their

model for analysis suggests using the family of power-divergence statistics, selecting the particular value of λ which is optimal under given conditions. For example, they indicate that Pearson's chi-square ($\lambda=1$) may be optimal when expected cell frequencies are near equal across a table, and when the number of table cells is at least 20, and G^2 is optimal for certain nonlocal alternatives with a limited number of near-zero probabilities. When the expected cell frequencies are unequal, they recommend $\lambda = 2/3$, specifying that Pearson's chi-square statistic can display serious bias for sparse tables with unequal cell probabilities.

For the conditions examined in this study, it would appear that a decision regarding the best statistic to apply in a given situation needs to be based on table size as well as sample size. (Skewness also impacted the results, by lessening power and increasing conservativeness, but this effect was consistent across statistical test.) In general, as Read and Cressie suggest, $\lambda = 2/3$ may be a good compromise solution when a researcher has little knowledge about possible alternative hypotheses. Conversely, a researcher may opt to use Pearson's test under large table, small sample conditions, the Read and Cressie power-divergence statistic under small table, small sample conditions, and either statistic with large samples.

Future research should include a more detailed investigation into sparseness. One aspect of this study which may limit the generalization of results concerns the manner in which data were generated. The data were simulated according to expected marginal proportions (e.g., equal marginals, slightly skewed, and highly skewed marginals), while the total table sample size was held near constant. For example, a 5 x 7 table with sample size of 256 will be sparser than a 2 x 7

table with the same total sample size. Thus, as table size increased, simultaneously sparseness also increased. A follow-up study could alter this data-generation design to provide more information about the interaction of sample size with table size.. Additionally, other approaches for modeling the alternative hypothesis could be simulated. While one reasonable alternative was modeled in the current study, a given contingency table can differ from the null in many ways, potentially affecting the selection of the optimal statistical test.

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Table 1
Empirical Estimates of Type I Error Rates by Table Size, Sample Size, Marginal Distribution and Nominal Alpha Level.

Nominal Alpha Level																												
.01							.05							.10														
Marginal Distribution							Marginal Distribution							Marginal Distribution														
Table Size	Sample Size	Equal			Slight Skew		Extreme Skew		Equal			Slight Skew		Extreme Skew		Equal			Slight Skew		Extreme Skew							
		Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC						
2 x 5	16	2	22	4	0	5	0	0	1	0	30	106	36	11	63	17	2	16	3	81	213	98	47	159	57	14	110	17
2 x 5	32	5	21	7	2	15	3	0	4	0	45	92	53	24	94	31	7	56	11	103	159	111	75	177	89	34	163	47
2 x 5	64	6	13	8	6	19	8	1	14	2	45	59	48	38	81	45	22	88	31	97	113	101	92	155	99	68	172	79
2 x 5	128	10	12	10	5	12	6	6	23	8	52	58	54	49	61	50	44	88	52	100	108	101	105	118	108	97	157	106
2 x 5	256	11	12	11	9	12	9	5	11	6	50	54	51	48	55	48	44	59	47	104	109	105	94	101	95	95	114	98
2 x 7	16	1	17	1	0	3	1	0	0	0	17	126	27	6	56	8	0	12	2	64	239	79	30	155	41	9	90	12
2 x 7	32	2	24	4	0	11	1	0	1	0	31	111	40	13	100	20	2	49	4	88	192	104	53	207	72	18	137	28
2 x 7	64	7	19	9	2	21	4	0	8	0	44	78	50	30	103	38	8	85	12	99	137	107	81	183	93	38	193	52
2 x 7	128	9	13	10	6	17	7	2	20	4	47	56	48	41	73	44	26	104	36	96	109	99	89	143	98	80	189	94
2 x 7	256	9	11	9	8	13	9	6	17	7	47	52	48	48	58	50	35	72	39	102	107	103	95	108	97	87	138	95
3 x 5	18	3	18	3	1	6	1	0	0	0	32	113	33	18	48	18	4	12	5	80	226	89	55	129	52	20	63	19
3 x 5	33	5	23	5	2	9	2	0	2	0	39	106	43	27	76	28	10	27	9	87	192	94	68	169	69	36	93	34
3 x 5	66	8	17	9	7	22	8	3	11	2	46	77	48	39	98	41	28	79	28	96	139	102	88	176	95	70	168	71
3 x 5	129	8	13	9	10	22	10	8	24	8	48	61	51	50	77	53	42	108	45	99	113	101	98	142	102	97	199	103
3 x 5	258	10	10	9	10	13	10	9	21	10	45	50	46	48	57	49	49	79	51	101	107	101	94	110	95	104	146	108
3 x 7	18	1	13	1	0	3	0	0	0	0	21	107	22	9	29	8	2	7	2	70	244	72	36	96	31	12	41	11
3 x 7	33	3	27	4	1	6	1	0	1	0	33	135	38	16	63	15	6	16	5	85	238	94	52	166	51	23	63	19
3 x 7	66	7	27	8	3	21	3	1	6	0	50	114	56	32	116	37	13	54	12	104	191	113	87	221	88	43	154	41
3 x 7	129	9	16	10	3	24	4	4	18	4	49	71	53	42	96	46	31	112	32	96	125	101	86	169	94	74	214	87
3 x 7	258	7	9	7	9	13	8	5	21	5	45	55	47	47	68	49	40	106	46	95	108	96	96	128	98	95	189	102
5 x 5	15	4	3	1	1	1	0	1	0	0	27	46	14	15	15	7	6	4	3	69	145	46	44	50	24	24	24	20
5 x 5	30	5	25	3	2	4	1	1	0	0	41	148	37	28	45	16	11	5	3	95	275	93	70	115	52	35	21	14
5 x 5	65	7	31	7	7	23	5	3	3	1	42	114	47	44	115	40	33	38	16	96	210	100	90	219	87	70	101	47
5 x 5	130	9	20	9	7	24	6	9	20	6	53	75	52	49	109	50	44	110	36	96	137	102	98	187	103	89	214	83
5 x 5	255	9	13	10	11	16	10	8	23	7	54	63	55	46	76	48	41	101	40	102	114	102	89	132	94	89	186	91
5 x 7	15	1	0	0	1	0	0	0	0	0	21	14	6	8	7	2	2	2	1	60	69	27	30	21	13	15	11	10
5 x 7	30	3	13	1	1	0	0	0	0	0	29	120	22	15	11	3	5	1	1	84	244	66	47	48	21	22	6	5
5 x 7	65	7	41	6	4	12	2	3	1	1	42	168	44	32	85	22	18	12	6	94	283	102	75	193	59	45	40	24
5 x 7	130	8	26	8	7	32	6	3	7	2	47	101	49	44	142	42	28	75	10	95	179	98	92	259	95	68	175	52
5 x 7	255	8	14	8	6	24	6	7	34	6	41	62	44	41	100	42	46	137	43	92	120	95	90	179	95	90	240	91

N.B. Estimates are based on 5000 samples of each condition. Type I error estimates have been multiplied by 1000 and rounded.

Table 2
Average Empirical Estimates of Statistical Power by Table Size, Sample Size, Marginal Distribution and Nominal Alpha Level.

Table Size	Sample Size	Nominal Alpha Level											
		.01						.05					
		Marginal Distribution						Marginal Distribution					
		Equal		Slight Skew		Extreme Skew		Equal		Slight Skew		Extreme Skew	
		CH12	LR	RC	CH12	LR	RC	CH12	LR	RC	CH12	LR	RC
2 x 5	16	18	31	5	6	3	3	129	145	57	76	28	40
2 x 5	32	12	141	61	81	28	32	296	311	247	279	155	174
2 x 5	64	343	349	340	356	292	301	489	495	486	498	440	472
2 x 5	128	537	540	530	536	517	531	634	636	638	642	636	645
2 x 5	256	673	674	674	675	674	677	730	730	730	731	730	732
2 x 7	16	3	9	7	8	1	1	71	95	73	82	17	22
2 x 7	32	75	93	58	78	9	19	229	256	203	231	87	124
2 x 7	64	291	305	271	287	179	220	451	461	429	444	377	397
2 x 7	128	499	503	486	492	463	477	611	614	599	605	588	602
2 x 7	256	654	655	653	655	650	654	716	717	718	719	714	719
3 x 5	10	19	19	7	6	2	1	101	107	55	55	21	22
3 x 5	33	89	90	49	47	13	11	225	236	162	163	80	69
3 x 5	66	265	267	213	215	115	108	421	425	378	385	283	280
3 x 5	129	468	468	455	456	389	388	582	583	574	577	542	548
3 x 5	258	634	633	634	634	630	633	703	704	706	707	701	705
4 x 7	18	9	8	4	4	1	0	65	67	40	41	12	12
4 x 7	33	51	52	28	26	6	4	168	173	121	119	54	47
4 x 7	64	215	214	177	177	103	94	370	375	333	340	261	253
4 x 7	129	426	423	417	421	381	381	539	539	539	545	509	512
4 x 7	258	605	603	603	602	592	594	685	685	681	683	675	678
5 x 5	15	7	2	3	1	1	1	49	29	27	13	11	6
5 x 5	30	32	26	20	10	10	4	121	115	87	60	53	26
5 x 5	65	140	139	121	107	78	50	283	287	254	245	196	150
5 x 5	130	361	360	335	329	302	278	407	409	466	466	438	427
5 x 5	255	549	547	529	524	511	505	640	641	621	621	608	606
5 x 7	15	4	0	1	0	0	0	36	16	18	7	6	3
5 x 7	30	24	13	13	5	6	1	101	80	62	32	37	17
5 x 7	65	127	121	100	73	56	25	267	268	226	195	163	99
5 x 7	130	346	344	321	310	285	242	477	478	456	452	425	388
5 x 7	255	531	533	528	524	511	500	629	628	626	626	610	605
5 x 7	15	4	0	1	0	0	0	36	16	18	7	6	3
5 x 7	30	24	13	13	5	6	1	101	80	62	32	37	17
5 x 7	65	127	121	100	73	56	25	267	268	226	195	163	99
5 x 7	130	346	344	321	310	285	242	477	478	456	452	425	388
5 x 7	255	531	533	528	524	511	500	629	628	626	626	610	605

Note: Estimates are based on 5000 samples of each condition. Power estimates have been multiplied by 1000 and rounded.

Table 3
Empirical Estimates of Statistical Power by Table Size, Sample Size, Effect Size, Marginal Distribution and Nominal Alpha Level.

Table Size	Effect Size	Nominal Alpha Level											
		.01						.05					
		Marginal Distribution						Marginal Distribution					
		Equal		Slight Skew		Extreme Skew		Equal		Slight Skew		Extreme Skew	
		Chi2	LR	Chi2	LR	Chi2	LR	Chi2	LR	Chi2	LR	Chi2	LR
2 x 5	0.1	3	...	5	0	...	0	39	...	47	15	2	...
2 x 5	0.3	7	...	14	3	...	0	92	...	108	39	54	...
2 x 5	0.5	44	...	73	12	...	14	256	...	280	117	153	...
2 x 5	0.1	8	...	13	2	...	3	63	...	69	34	12	...
2 x 5	0.3	52	...	63	23	...	33	208	...	226	151	179	...
2 x 5	0.5	313	...	346	159	...	208	83	...	638	555	612	...
2 x 5	0.1	17	...	19	11	...	13	81	...	86	69	76	...
2 x 5	0.3	208	...	218	174	...	196	59	...	456	420	446	...
2 x 5	0.5	805	...	811	855	...	860	815	...	942	968	973	...
2 x 5	0.1	36	...	38	27	...	31	116	...	118	120	124	...
2 x 5	0.3	578	...	584	564	...	578	536	...	789	795	801	...
2 x 5	0.5	997	...	997	1000	...	1000	1000	...	1000	1000	1000	...
2 x 5	0.1	73	...	74	71	...	73	57	...	203	201	205	...
2 x 5	0.3	946	...	947	952	...	953	964	...	980	988	989	...
2 x 5	0.5	1000	...	1000	1000	...	1000	1000	...	1000	1000	1000	...
2 x 7	0.1	0	...	1	0	...	0	17	...	28	10	13	...
2 x 7	0.3	2	...	4	2	...	3	48	...	69	48	57	...
2 x 7	0.5	8	...	21	19	...	22	3	...	188	161	176	...
2 x 7	0.1	4	...	7	3	...	4	45	...	56	20	28	...
2 x 7	0.3	28	...	39	20	...	30	154	...	183	121	148	...
2 x 7	0.5	194	...	233	151	...	200	25	...	528	468	518	...
2 x 7	0.1	12	...	15	4	...	7	71	...	78	47	57	...
2 x 7	0.3	151	...	169	129	...	145	39	...	404	347	372	...
2 x 7	0.5	719	...	730	680	...	710	497	...	902	893	904	...
2 x 7	0.1	28	...	29	18	...	20	9	...	107	87	96	...
2 x 7	0.3	481	...	492	451	...	467	381	...	735	711	721	...
2 x 7	0.5	989	...	989	989	...	989	999	...	999	999	999	...
2 x 7	0.1	51	...	53	56	...	58	45	...	176	180	184	...
2 x 7	0.3	911	...	912	902	...	906	904	...	974	973	974	...
2 x 7	0.5	1000	...	1000	1000	...	1000	1000	...	1000	1000	1000	...
2 x 7	0.1	0	...	1	0	...	0	17	...	28	10	13	...
2 x 7	0.3	2	...	4	2	...	3	48	...	69	48	57	...
2 x 7	0.5	8	...	21	19	...	22	3	...	188	161	176	...
2 x 7	0.1	4	...	7	3	...	4	45	...	56	20	28	...
2 x 7	0.3	28	...	39	20	...	30	154	...	183	121	148	...
2 x 7	0.5	194	...	233	151	...	200	25	...	528	468	518	...
2 x 7	0.1	12	...	15	4	...	7	71	...	78	47	57	...
2 x 7	0.3	151	...	169	129	...	145	39	...	404	347	372	...
2 x 7	0.5	719	...	730	680	...	710	497	...	902	893	904	...
2 x 7	0.1	28	...	29	18	...	20	9	...	107	87	96	...
2 x 7	0.3	481	...	492	451	...	467	381	...	735	711	721	...
2 x 7	0.5	989	...	989	989	...	989	999	...	999	999	999	...
2 x 7	0.1	51	...	53	56	...	58	45	...	176	180	184	...
2 x 7	0.3	911	...	912	902	...	906	904	...	974	973	974	...
2 x 7	0.5	1000	...	1000	1000	...	1000	1000	...	1000	1000	1000	...

Table 3 (Continued)
Empirical Estimates of Statistical Power by Table Size, Sample Size, Effect Size, Marginal Distribution and Nominal Alpha Level.

			.01						.05						.10														
Table Size	Sample Size	Effect Size	Marginal Distribution						Marginal Distribution						Marginal Distribution														
			Equal			Slight Skew			Extreme Skew			Equal			Slight Skew			Extreme Skew			Equal			Slight Skew			Extreme Skew		
			Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC
3 x 5	18	0.1	3	...	3	2	...	2	0	...	0	37	...	41	18	...	18	7	...	8	90	...	98	57	...	55	26	...	26
3 x 5	18	0.3	9	...	9	1	...	1	2	...	1	71	...	76	40	...	40	17	...	17	155	...	165	110	...	103	56	...	54
3 x 5	18	0.5	45	...	44	17	...	16	4	...	3	195	...	204	108	...	108	40	...	40	340	...	351	242	...	228	114	...	109
3 x 5	33	0.1	8	...	8	3	...	3	1	...	1	55	...	62	32	...	33	13	...	10	120	...	129	87	...	91	65	...	44
3 x 5	33	0.3	43	...	44	21	...	20	7	...	6	152	...	163	114	...	114	54	...	49	264	...	280	221	...	226	136	...	130
3 x 5	33	0.5	216	...	218	123	...	119	31	...	27	467	...	483	341	...	341	172	...	148	611	...	625	496	...	500	315	...	305
3 x 5	66	0.1	12	...	13	10	...	11	5	...	5	68	...	73	62	...	67	42	...	40	135	...	143	121	...	129	93	...	94
3 x 5	66	0.3	139	...	142	124	...	128	65	...	61	344	...	350	312	...	321	239	...	237	471	...	476	456	...	469	387	...	390
3 x 5	66	0.5	644	...	646	504	...	507	274	...	259	850	...	853	761	...	768	568	...	563	916	...	918	854	...	860	718	...	718
3 x 5	129	0.1	26	...	26	24	...	24	16	...	16	106	...	106	90	...	94	80	...	84	183	...	185	164	...	169	155	...	164
3 x 5	129	0.3	402	...	400	393	...	393	330	...	330	646	...	646	643	...	646	601	...	611	756	...	759	761	...	766	733	...	741
3 x 5	129	0.5	976	...	977	947	...	951	822	...	818	995	...	996	990	...	991	946	...	949	998	...	998	996	...	997	973	...	975
3 x 5	258	0.1	54	...	54	51	...	53	43	...	42	162	...	165	163	...	165	151	...	156	269	...	271	263	...	267	254	...	259
3 x 5	258	0.3	848	...	846	851	...	850	848	...	859	948	...	948	954	...	955	953	...	959	973	...	972	978	...	979	980	...	983
3 x 5	258	0.5	1000	...	1000	1000	...	1000	998	...	998	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000
3 x 7	18	0.1	1	...	1	1	...	1	0	...	0	25	...	27	10	...	10	2	...	2	72	...	74	39	...	32	12	...	11
3 x 7	18	0.3	5	...	5	2	...	2	0	...	0	46	...	47	28	...	28	8	...	9	120	...	122	80	...	68	35	...	31
3 x 7	18	0.5	21	...	18	9	...	8	2	...	1	124	...	126	83	...	84	25	...	25	257	...	258	202	...	175	92	...	78
3 x 7	33	0.1	6	...	6	2	...	2	0	...	0	38	...	42	19	...	18	7	...	5	92	...	101	61	...	60	29	...	25
3 x 7	33	0.3	22	...	23	9	...	9	2	...	1	117	...	120	69	...	67	31	...	25	212	...	227	157	...	156	92	...	81
3 x 7	33	0.5	125	...	126	72	...	66	17	...	12	350	...	357	275	...	272	125	...	110	499	...	514	432	...	429	272	...	256
3 x 7	66	0.1	12	...	13	7	...	7	2	...	1	65	...	71	47	...	50	22	...	21	129	...	139	102	...	108	63	...	61
3 x 7	66	0.3	109	...	110	68	...	70	28	...	25	274	...	284	276	...	236	147	...	140	411	...	423	360	...	377	275	...	267
3 x 7	66	0.5	524	...	518	455	...	455	278	...	257	770	...	771	726	...	733	615	...	598	850	...	858	839	...	849	779	...	770
3 x 7	129	0.1	17	...	17	17	...	19	8	...	8	77	...	80	77	...	84	56	...	58	149	...	153	147	...	160	118	...	126
3 x 7	129	0.3	318	...	311	306	...	313	218	...	215	555	...	553	559	...	569	485	...	491	686	...	686	689	...	698	629	...	641
3 x 7	129	0.5	944	...	940	929	...	932	917	...	919	986	...	985	981	...	982	986	...	987	995	...	995	997	...	993	995	...	995
3 x 7	258	0.1	44	...	45	37	...	38	33	...	33	147	...	150	135	...	140	125	...	132	233	...	233	223	...	228	214	...	224
3 x 7	258	0.3	771	...	763	771	...	767	744	...	749	908	...	905	908	...	908	899	...	903	952	...	951	951	...	949	945	...	948
3 x 7	258	0.5	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000	1000	...	1000

Table 3 (Continued)
Empirical Estimates of Statistical Power by Table Size, Sample Size, Effect Size, Marginal Distribution and Nominal Alpha Level.

Table Size		Nominal Alpha Level											
		.01						.05					
		Marginal Distribution						Marginal Distribution					
Effect Size	Sample Size	Equal			Slight Skew			Equal			Slight Skew		
		Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC	Chi2	LR	RC
0.1	15	3	...	1	2	...	1	0	...	0	29	...	16
0.3	15	4	...	1	1	...	0	1	...	1	40	...	24
0.5	15	14	...	4	6	...	2	2	...	1	78	...	46
0.1	30	7	...	5	4	...	1	1	...	0	45	...	41
0.3	30	20	...	15	11	...	3	5	...	3	91	...	87
0.5	30	70	...	50	46	...	26	25	...	9	228	...	218
0.1	65	9	...	9	11	...	9	5	...	1	56	...	59
0.3	65	64	...	64	59	...	49	32	...	19	197	...	200
0.5	65	347	...	345	294	...	264	197	...	131	595	...	603
0.1	130	15	...	16	15	...	14	11	...	8	70	...	73
0.3	130	235	...	228	207	...	197	172	...	163	448	...	448
0.5	130	834	...	836	782	...	776	723	...	683	944	...	946
0.1	255	33	...	33	29	...	27	27	...	24	110	...	111
0.3	255	614	...	608	564	...	551	513	...	498	811	...	811
0.5	255	999	...	999	995	...	995	994	...	994	1000	...	1000
0.1	15	1	...	0	0	...	0	0	...	0	22	...	7
0.3	15	3	...	0	0	...	0	0	...	0	29	...	11
0.5	15	7	...	1	3	...	1	1	...	1	58	...	25
0.1	30	6	...	2	1	...	0	1	...	0	43	...	30
0.3	30	16	...	8	6	...	3	2	...	0	71	...	19
0.5	30	50	...	30	31	...	13	14	...	3	190	...	155
0.1	65	11	...	11	6	...	2	53	...	56
0.3	65	59	...	55	43	...	27	18	...	5	179	...	180
0.5	65	310	...	297	252	...	190	148	...	69	568	...	567
0.1	130	14	...	14	15	...	13	8	...	5	65	...	69
0.3	130	191	...	189	176	...	160	136	...	91	420	...	420
0.5	130	835	...	829	772	...	758	710	...	630	947	...	946
0.1	255	29	...	30	25	...	24	22	...	17	100	...	104
0.3	255	583	...	570	564	...	552	514	...	487	787	...	782
0.5	255	998	...	998	995	...	995	996	...	995	999	...	999

Power estimates are based on 5000 samples of each condition. Power estimates have been multiplied by 1000 and rounded.

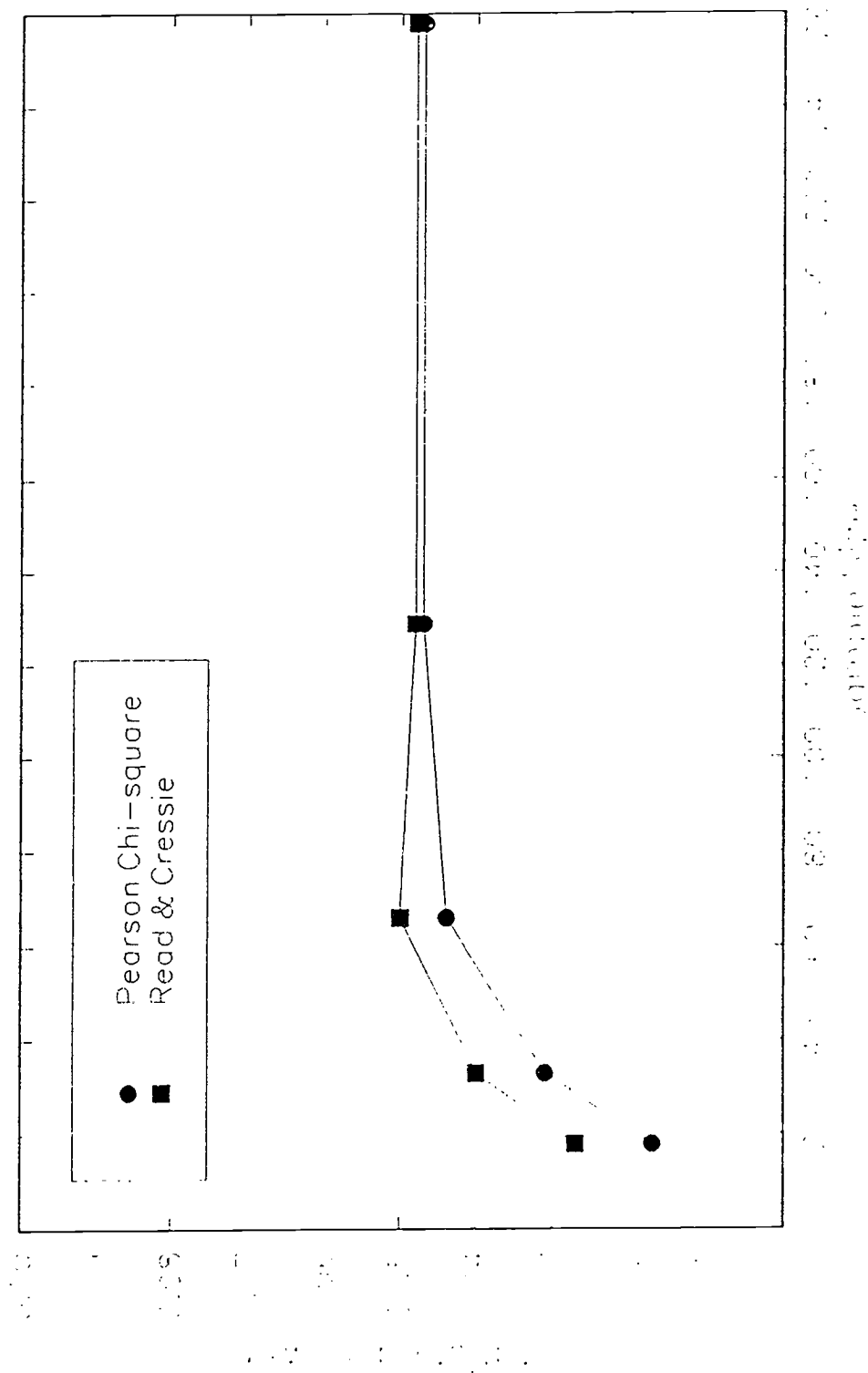
Table 4

Number and Percent of Conditions with Differences in Empirical Power Estimates by Table Size.

Table Size	Nominal Alpha = .01			Nominal Alpha = .05			Nominal Alpha = .10		
	Power Differences			Power Differences			Power Differences		
	RC>Chi2	No Diff	Chi2>RC	RC>Chi2	No Diff	Chi2>RC	RC>Chi2	No Diff	Chi2>RC
2 x 5	33 73%	12 27%	0 0%	37 82%	8 18%	0 0%	33 73%	12 27%	0 0%
2 x 7	33 73%	12 27%	0 0%	38 84%	7 16%	0 0%	36 80%	9 20%	0 0%
3 x 5	12 27%	18 40%	15 33%	26 58%	13 29%	6 13%	30 67%	6 13%	9 20%
3 x 7	12 27%	18 40%	15 33%	25 56%	8 18%	12 27%	20 44%	7 16%	18 40%
5 x 5	2 4%	8 18%	35 78%	8 18%	4 9%	33 73%	12 27%	6 13%	27 60%
5 x 7	1 2%	9 20%	35 78%	5 11%	4 9%	36 80%	10 22%	3 7%	32 71%

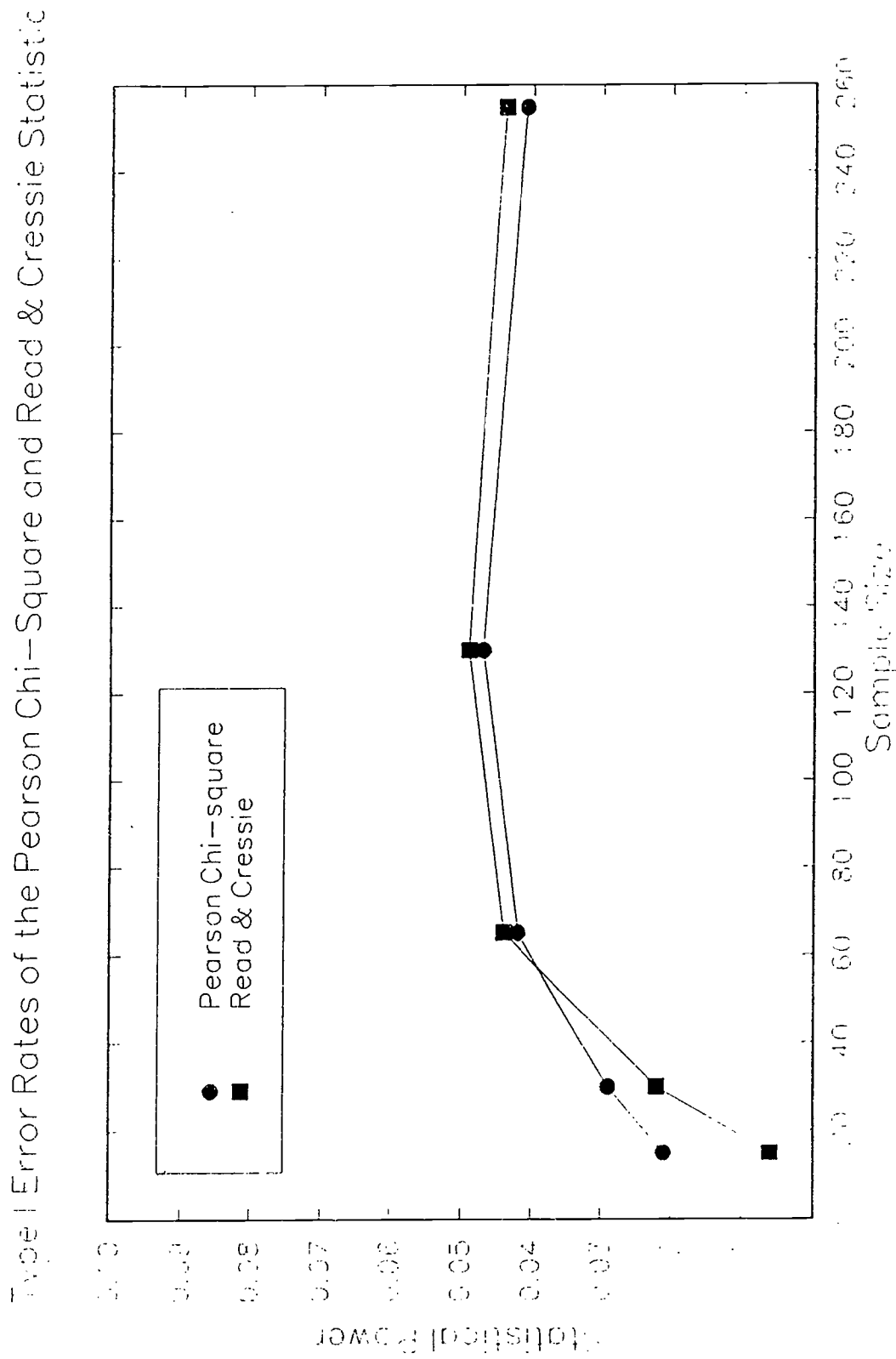
Figure 1

Type Error Rates of the Pearson Chi-Square and Read & Cressie Statistic



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Figure 2



Tables, Equal Marginals, Nominal Alpha = .05

Figure 3
Power of the Pearson Chi-Square and Read & Cressie Statistic

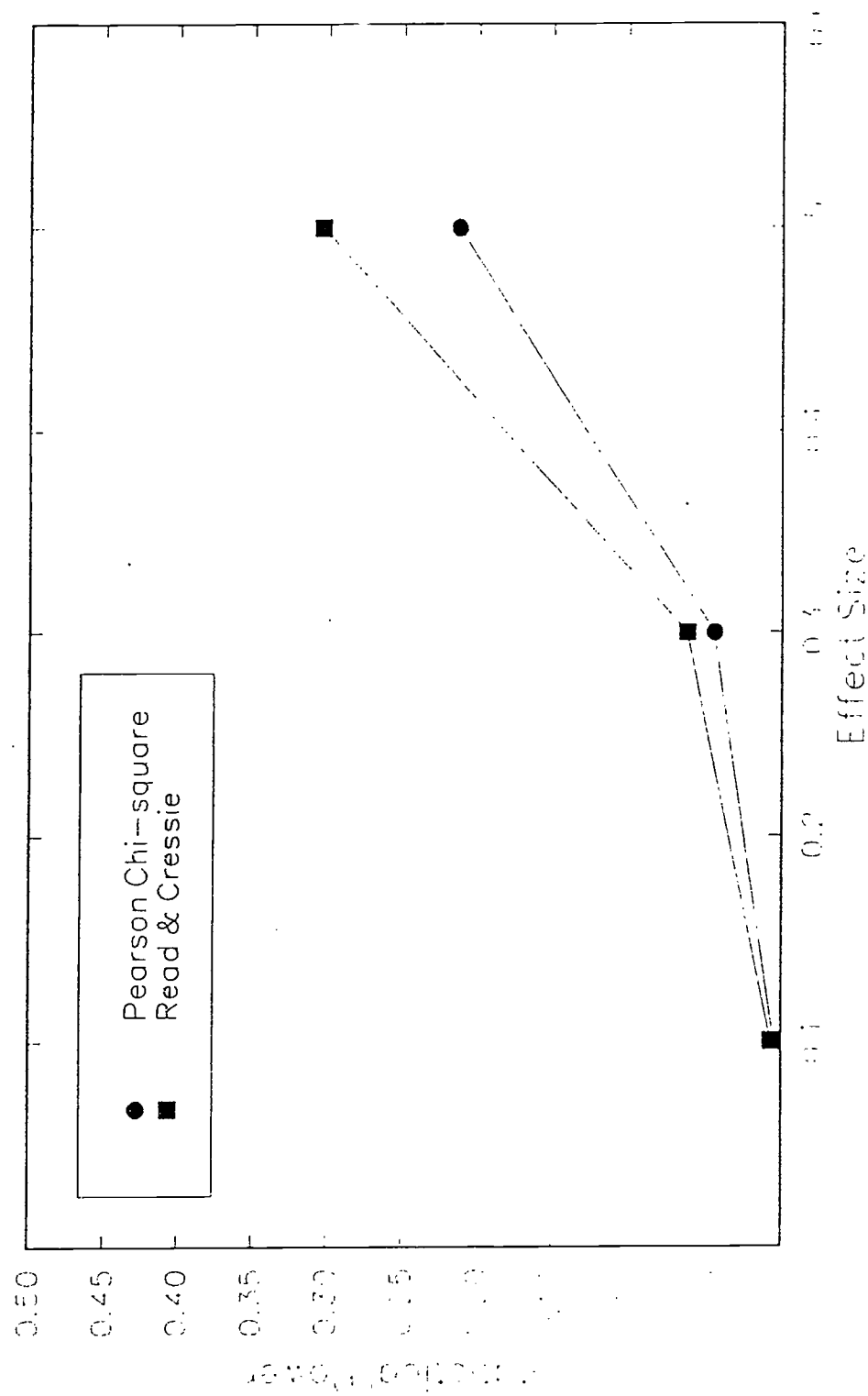
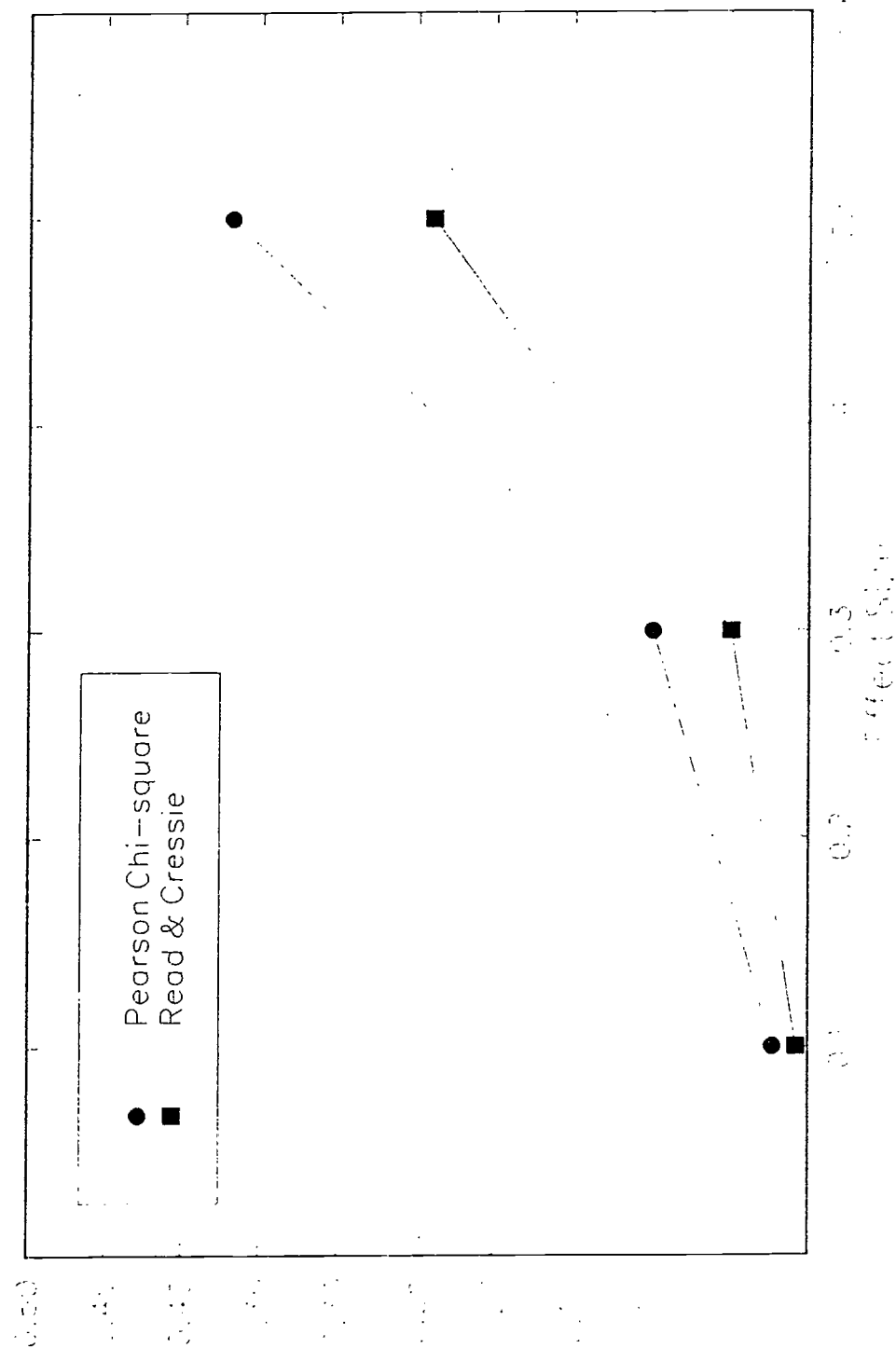


Figure 3. Power of the Pearson Chi-Square and Read & Cressie Statistic

Figure 4
Power of the Pearson Chi-Square and Read & Cressie Statistic



Power of the Pearson Chi-Square and Read & Cressie Statistic